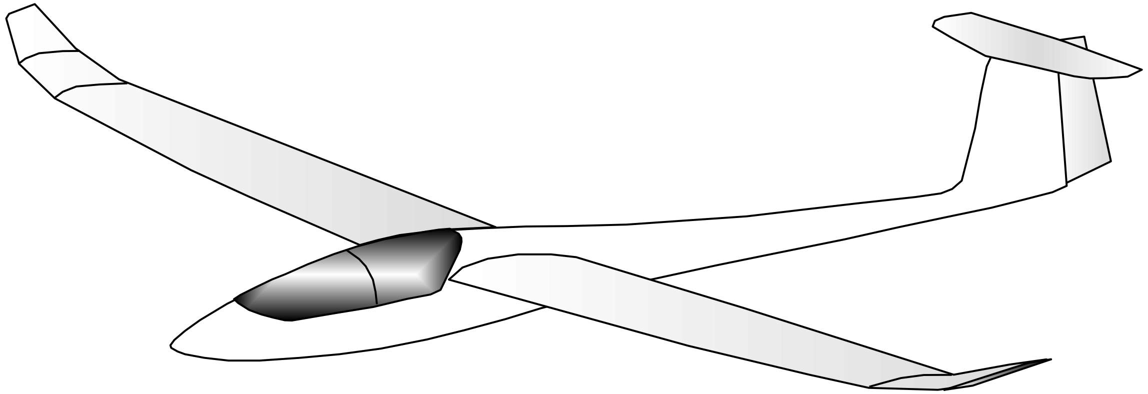
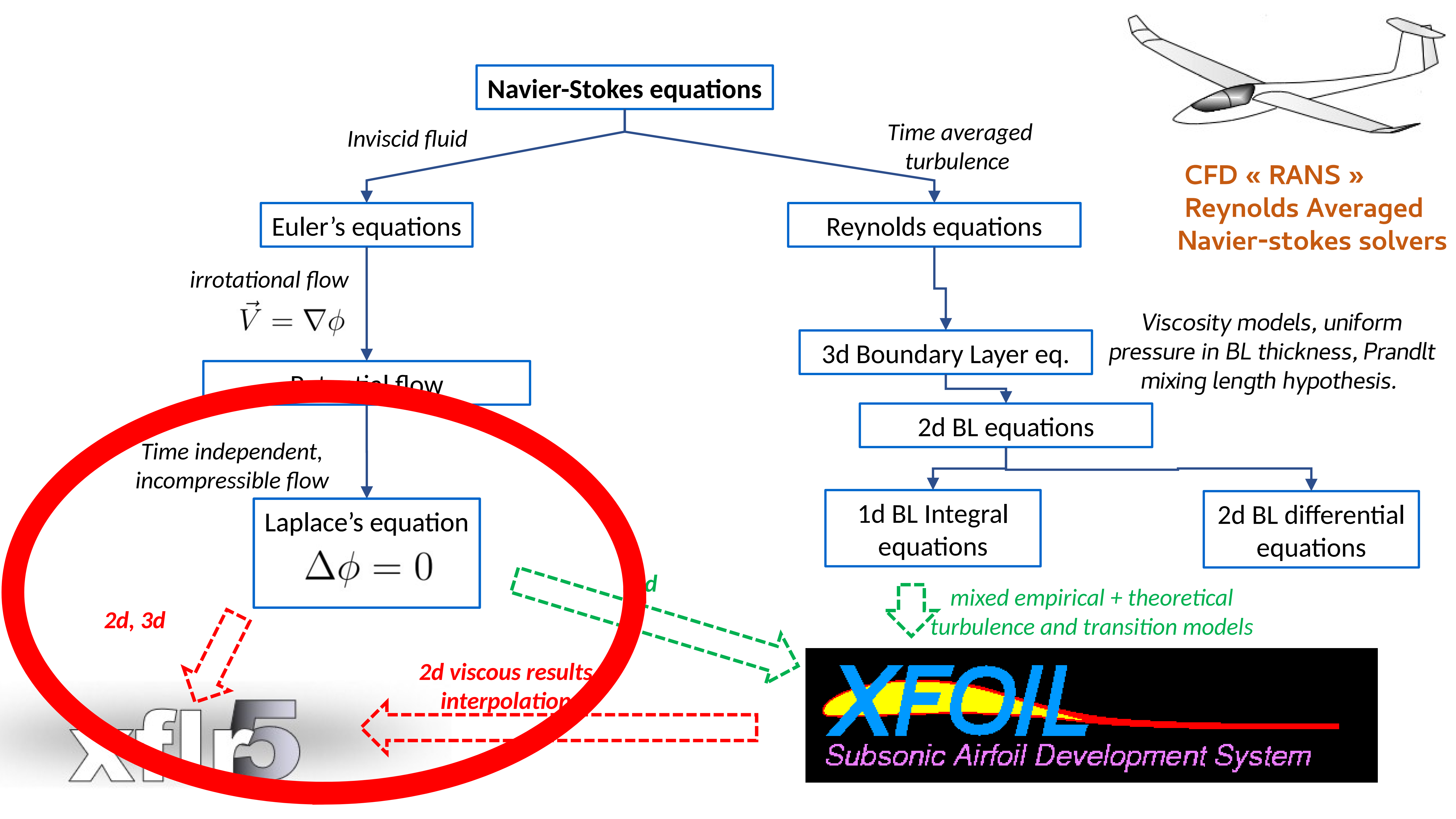


Why does a plane fly: the inviscid potential flow



Navier-Stokes equations
Inviscid fluid
Time averaged turbulence
CFD « RANS »
Reynolds Averaged
Navier-stokes solvers
Euler's equations
Reynolds equations
irrotational flow
$\vec{V} = \nabla\phi$
3d Boundary Layer eq.
Viscosity models, uniform pressure in BL thickness, Prandlt mixing length hypothesis.
flow
2d BL equations
Time independent, incompressible flow
Laplace's equation
$\Delta\phi = 0$
1d BL Integral equations
2d BL differential equations
mixed empirical + theoretical turbulence and transition models
2d, 3d
2d viscous results interpolation
XFOIL
Subsonic Airfoil Development System
xflr5

Potential flow: Scope of validity

Viscosity is omitted ⇒ The theory will be preferably applied where the viscous effects are negligible.

⇓ ⇓

The Reynolds number can be seen as a measure of the ratio of inertia forces to viscous forces ⇒ The theory will be preferably applied to high Reynolds number flows

Flow separation on an airfoil is a viscous effect ⇒ The theory will be preferably applied at low angles of attack without flow separation

Solving Laplace's equation

The potential is required to be such that
$\Delta\phi = 0$ everywhere in the flow
and to satisfy boundary conditions representing
the problem at hand.

Solving Laplace's equation

The idea:

If ϕ_1 and ϕ_2 are solutions of Laplace's equation, i.e.

$$\Delta\phi_1 = 0$$

$$\Delta\phi_2 = 0$$

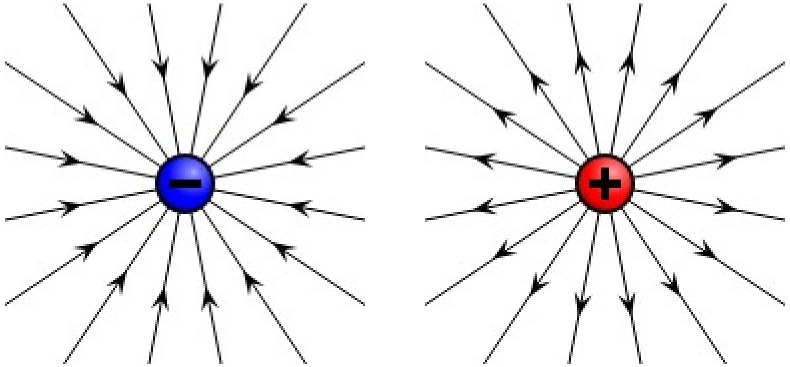
So is any linear combination of ϕ_1 and ϕ_2

$$\lambda\Delta\phi_1 + \mu\Delta\phi_2 = 0$$

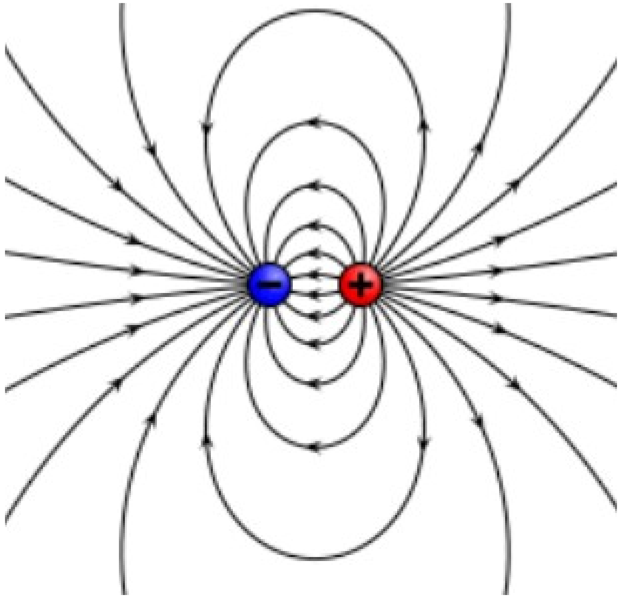
Solving Laplace's equation

In electrostatics, the elementary solutions are

the point charge



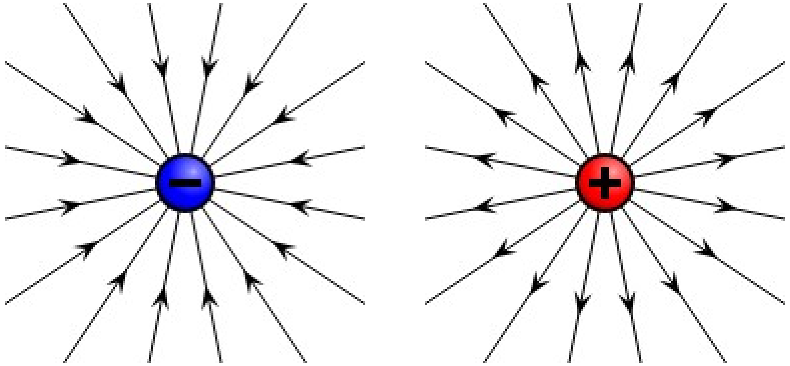
the doublet



Solving Laplace's equation

In fluid dynamics, the fundamental solutions are

the source and sink



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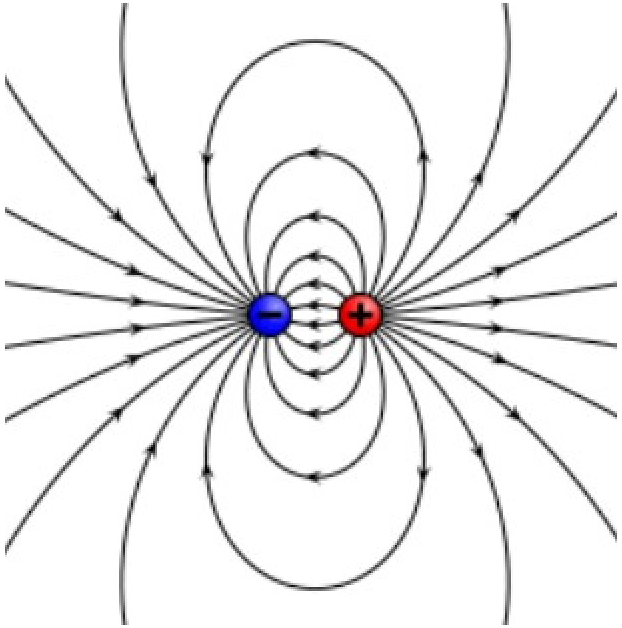
the doublet



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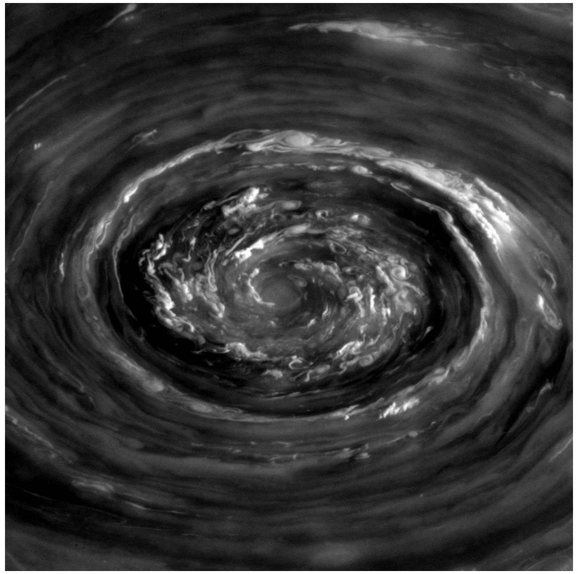
the vortex



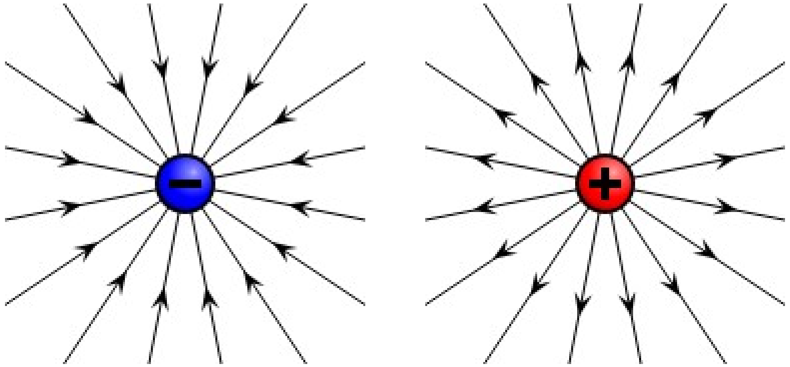
NASA / JPL-Caltech / Space Science Institute - http://www.ciclops.org/view/7436/Saturn_Rev_175_Raw_Preview_2

Solving Laplace's equation

The vortex strength is called the "circulation" Γ

$$V_\theta = -\frac{\Gamma}{2\pi\, r}$$

the source and sink



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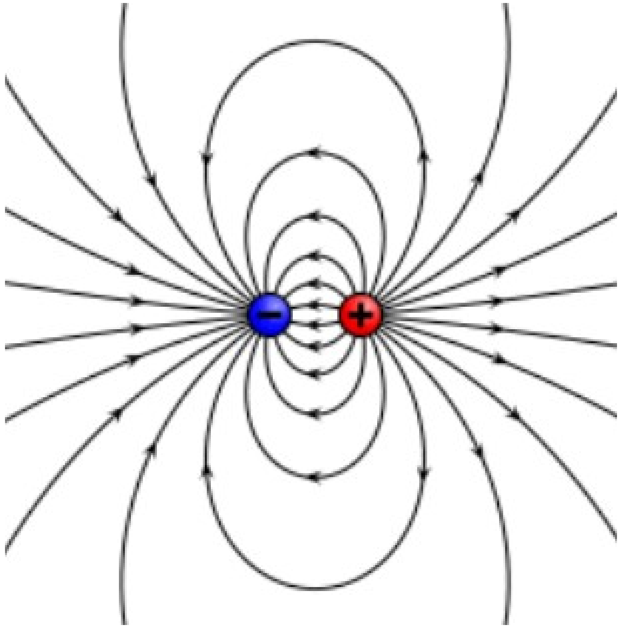
the doublet



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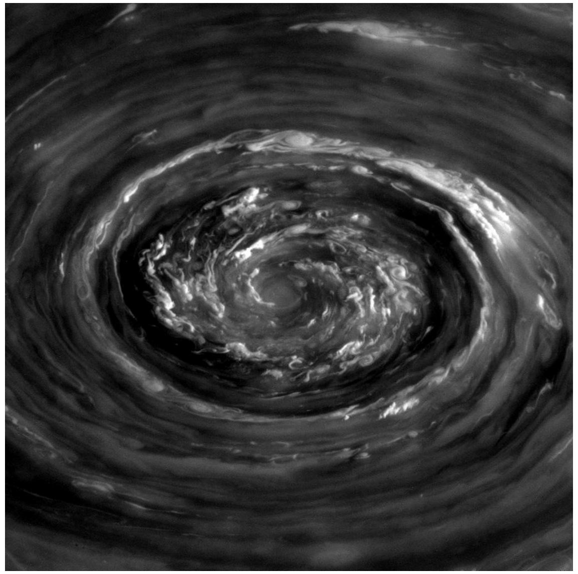
the vortex



NASA / JPL-Caltech / Space Science Institute - http://www.ciclops.org/view/7436/Saturn_Rev_175_Raw_Preview_2

Solving Laplace's equation

The Boundary Conditions can be of
the Dirichlet and Neumann types

Dirichlet BC: $\phi = \phi_0$ at specified locations in the fluid
Neumann BC: $\nabla\phi = \vec{V}_0$ on surfaces

Mixed BC are possible

Solving Laplace's equation

Dirichlet BC:

Mathematical concept, no simple physical interpretation

Neumann BC:

- is the specification of the velocity vector on given surfaces
- for solid surfaces, this is $\vec{V}.\vec{n} = 0$, i.e. the velocity is tangent to the surface

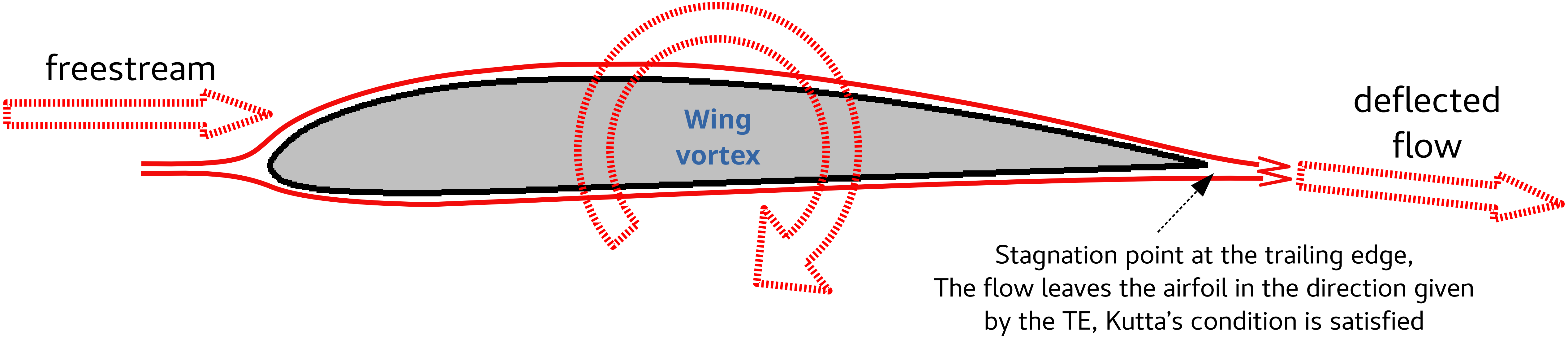
Solving Laplace's equation

The idea is to

1. Search for a solution which is a linear combination of N elementary solutions
2. Define N appropriate boundary conditions
3. Include Kutta's condition in the case of the panel method
4. Solve the problem of N equations = the BC, for N variables = the elementary solutions

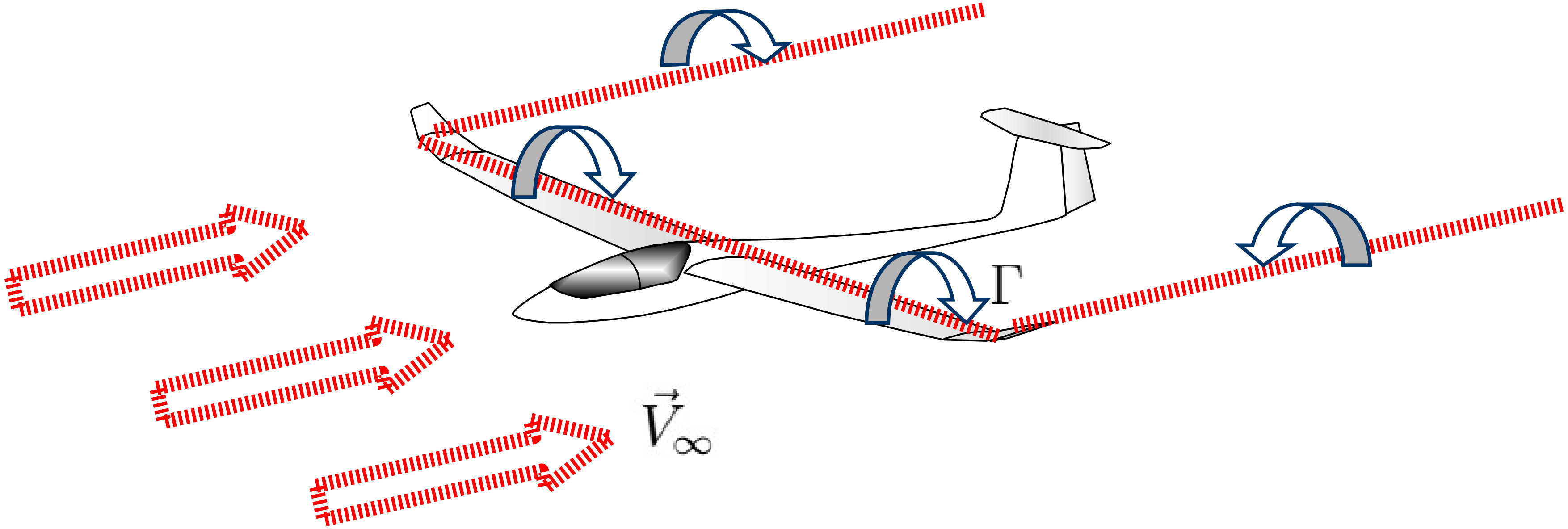
Kutta's condition

- A wing produces lift by deflecting the flow downwards
- It can be shown that this occurs if a vortex forms around the wing and adds its velocity field to the freestream velocity.
- Kutta's condition states that, to have physical sense, the vortex must be such that the total flow, i.e. freestream + vortex, leaves the wing smoothly at the trailing edge



A wing is a vortex generator

A wing produces lift by deflecting the freestream flow downwards

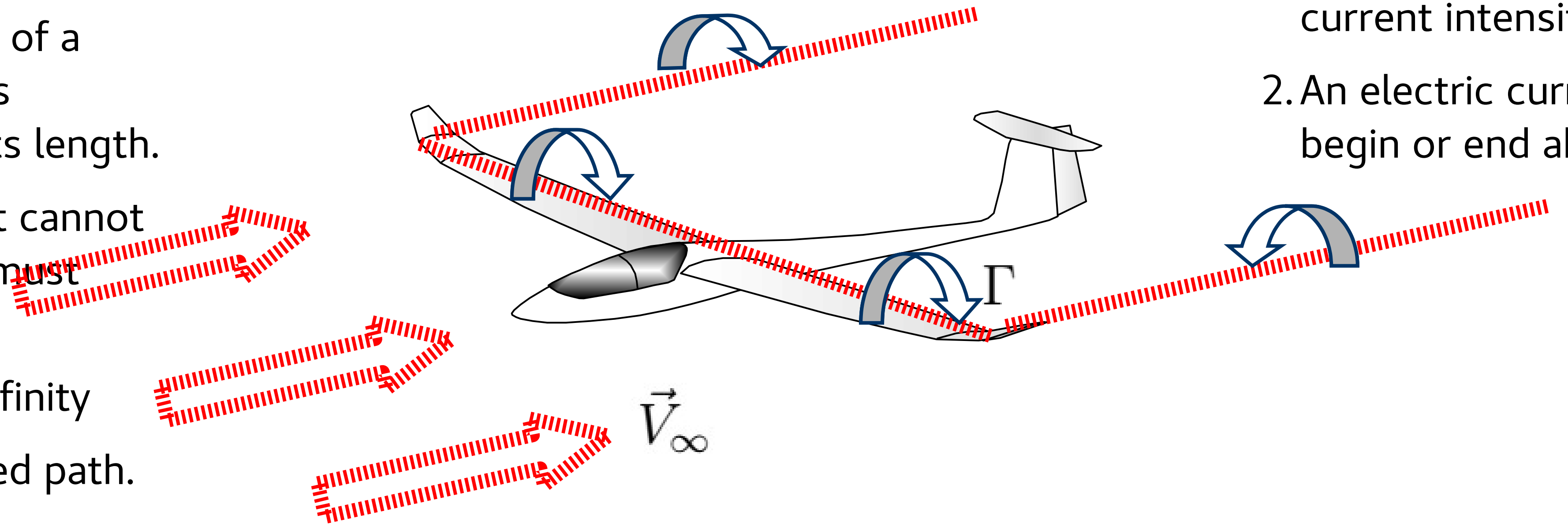


A wing is a vortex generator

Fluids

Helmholtz theorems

1. The circulation Γ of a vortex filament is constant along its length.
2. A vortex filament cannot end in a fluid; it must either:
   i. extend to infinity
   ii. form a closed path.



Electromagnetism

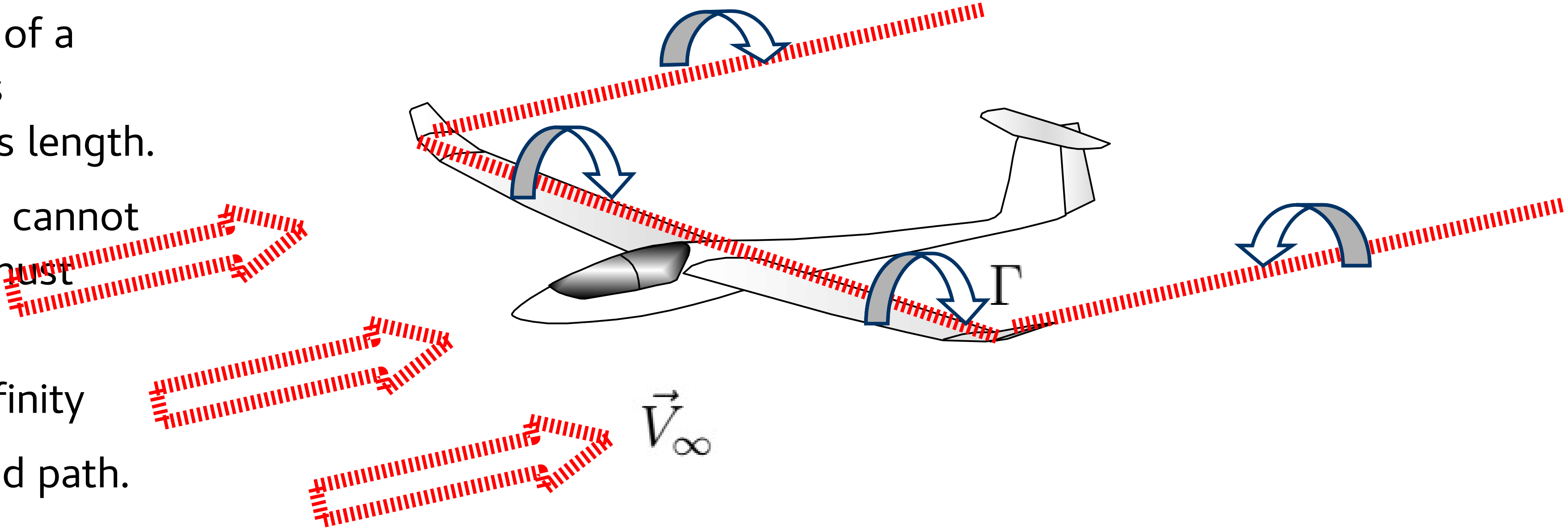
1. Along an electric wire, the current intensity is constant.
2. An electric current cannot begin or end abruptly.

A wing is a vortex generator

Fluids

Helmholtz theorems

1. The circulation Γ of a vortex filament is constant along its length.
2. A vortex filament cannot end in a fluid; it must either:
   i. extend to infinity
   ii. form a closed path.



- In 3d, this implies the existence of tip vortices, lost kinetic energy in the cross-flow, and induced drag
- In 2d, no tip vortices and no induced drag; this is the reason why the drag calculate by XFoil is pure viscous/profile drag

A wing is a vortex generator

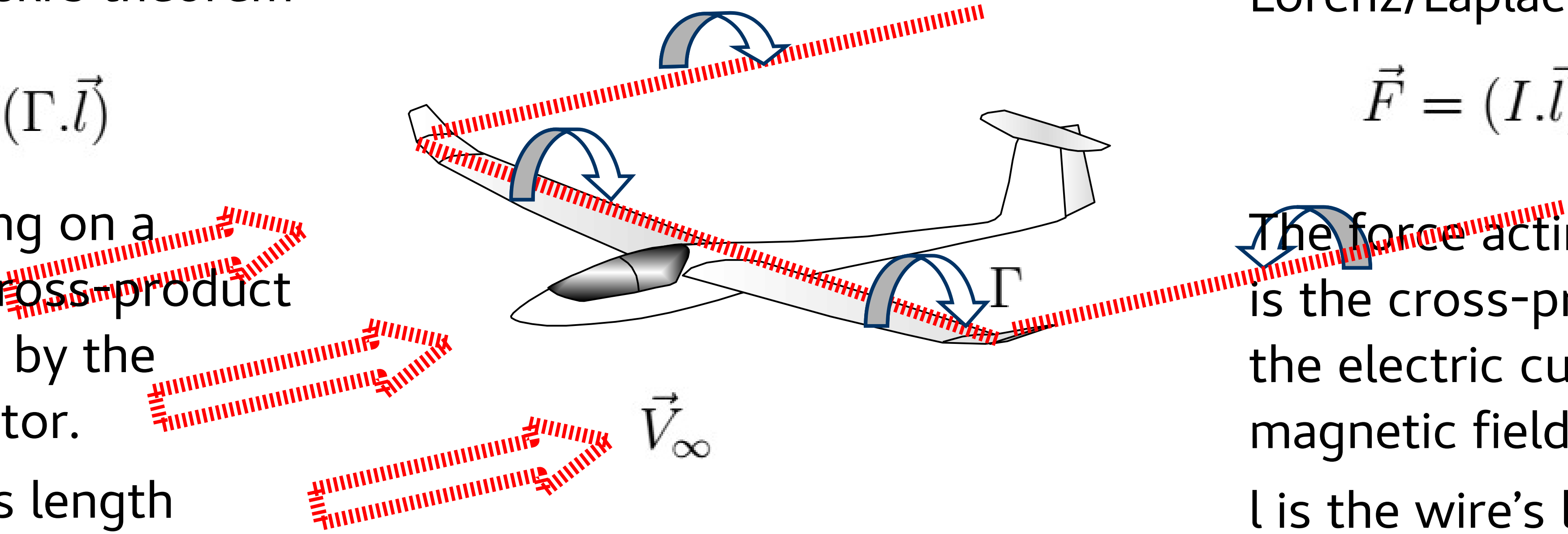
Fluids

Kutta-Joukowski's theorem

$$\vec{F} = \rho \vec{V}_\infty \otimes (\Gamma.\vec{l})$$

The force acting on a vortex is the cross-product of the velocity by the circulation vector.

l is the vortex's length



Electromagnetism

Lorenz/Laplace force

$$\vec{F} = (I.\vec{l}) \otimes \vec{B}$$

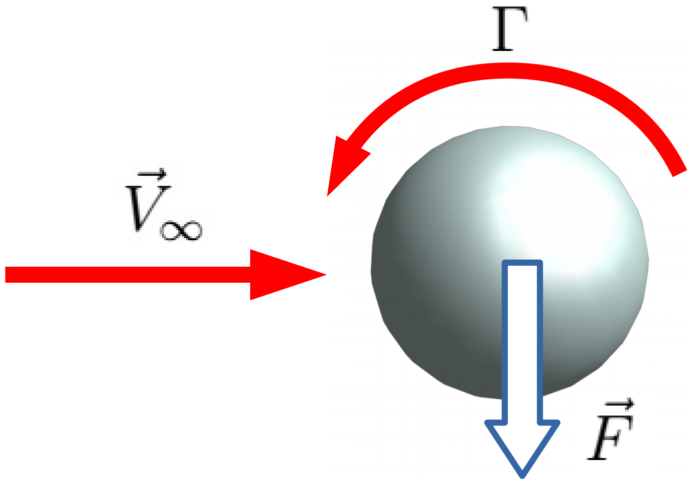
The force acting on a wire is the cross-product of the electric current by the magnetic field.

l is the wire's length

Similarly to the force on an electric wire in a magnetic field, a freestream flow creates on a vortex a radial force normal to the velocity vector

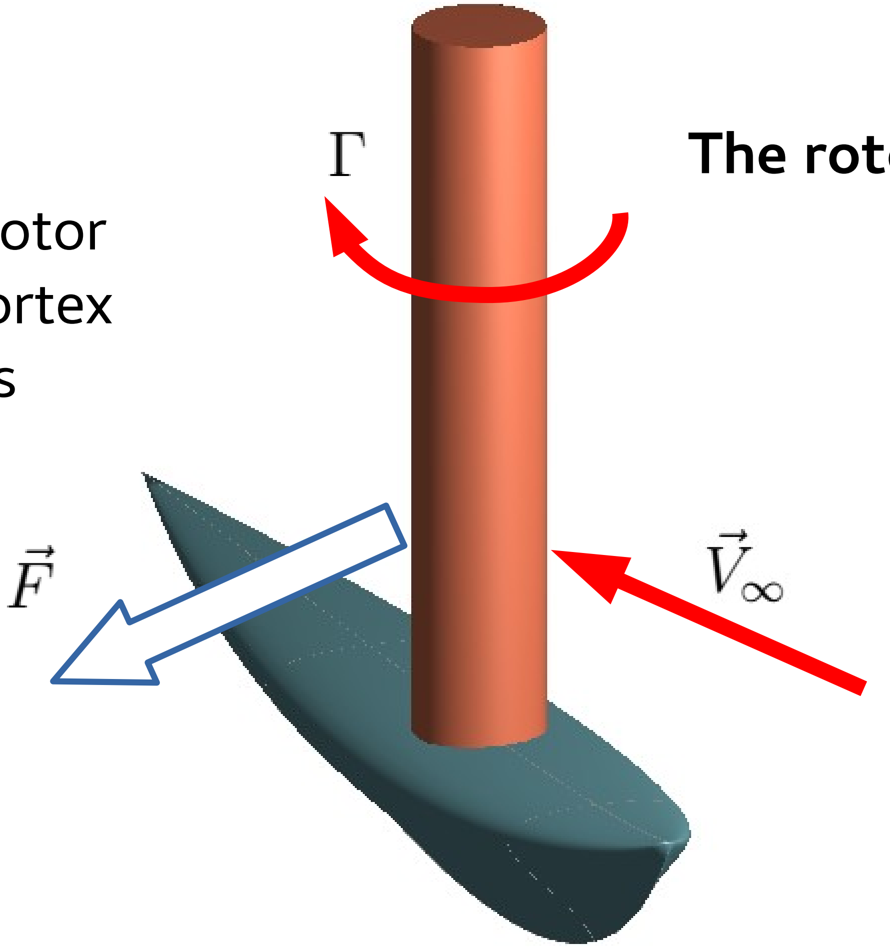
The Kutta-Joukowski theorem

The topspin ball



The ball and rotor mast act as vortex generators

The rotor boat



The Magnus effect is an example of the Kutta-Joukowski theorem

A wing is a vortex generator

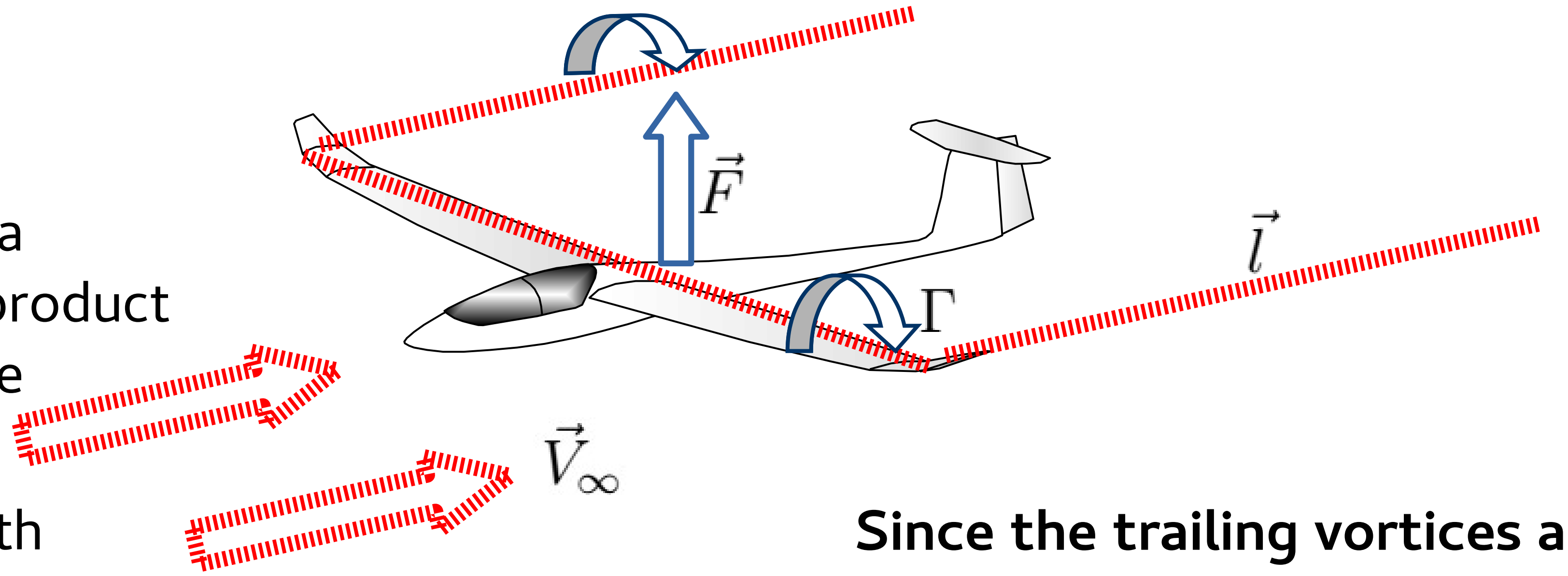
Fluids

Kutta-Joukowski's theorem

$$\vec{F} = \rho \vec{V}_\infty \otimes (\Gamma.\vec{l})$$

The force acting on a vortex is the cross-product of the velocity by the circulation vector.

l is the vortex's length



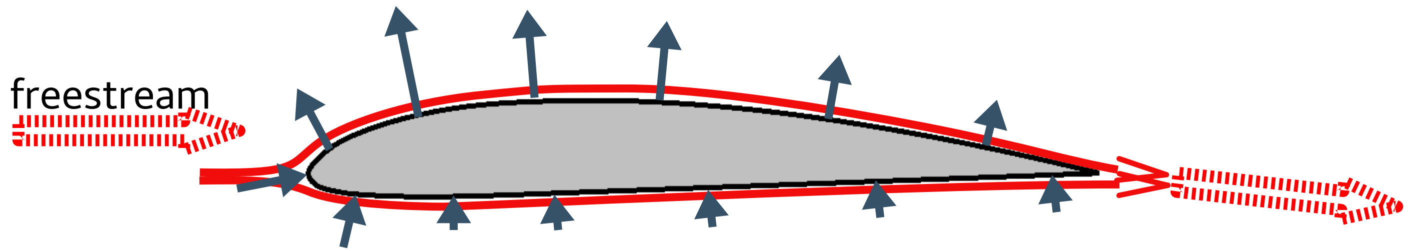
Since the trailing vortices are aligned with the free-stream,

$$\vec{F} = \vec{0}$$

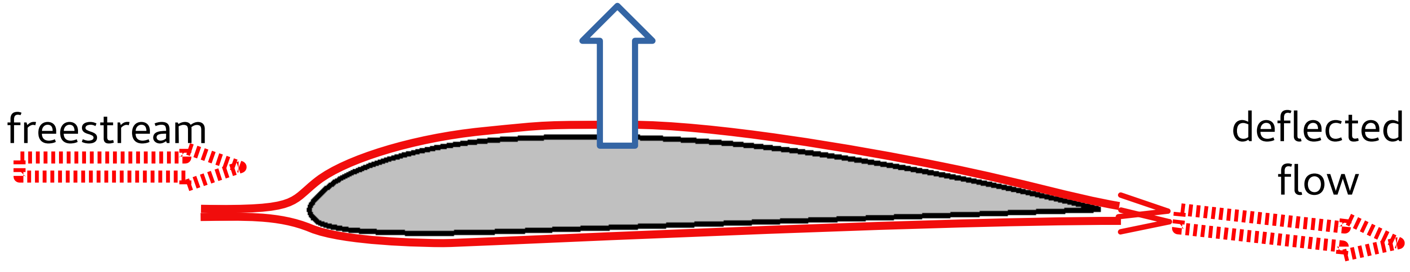
"The wake carries no load"

Why does a wing produce lift

The CFD point of view:
pressure forces

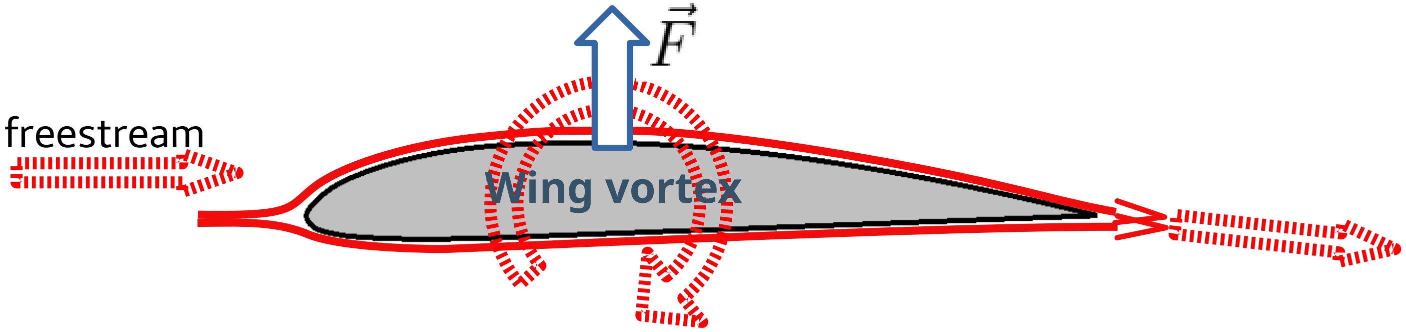


The mechanical point of view:
Action and reaction



$$\vec{F} = \frac{d}{dt}(m.\vec{V})$$

The mathematical point of view:
Kutta-Joukowski theorem

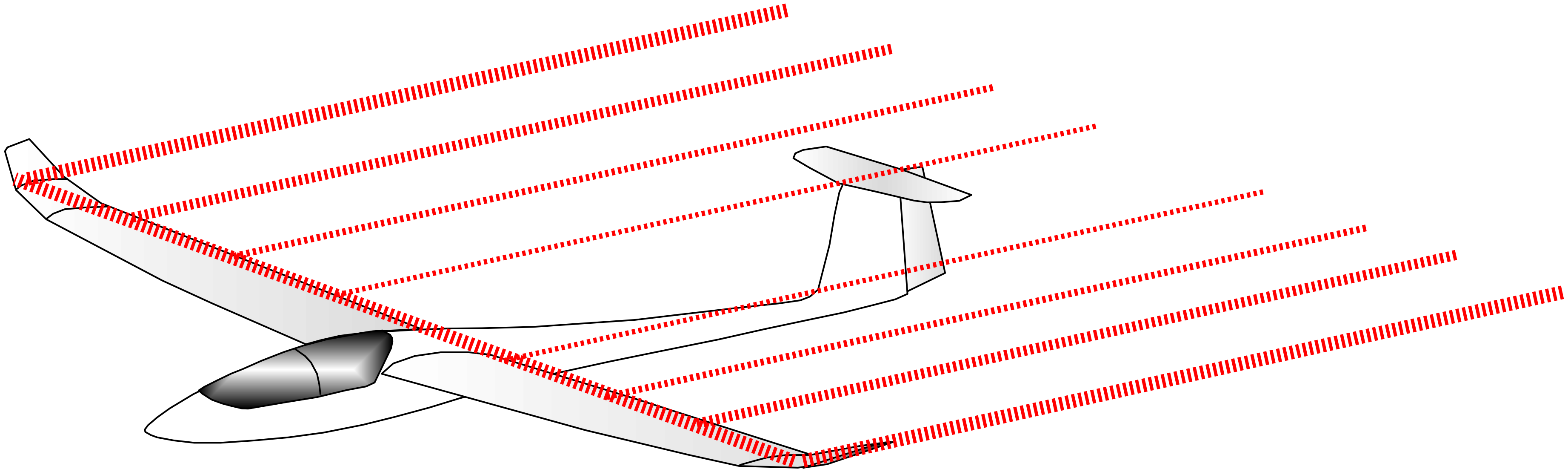


Solving Laplace's equation

3D	Elementary solutions	Boundary Condition
Lifting Line Theory (LLT)	Vortices	Neumann
Vortex Lattice Method (VLM)	Vortices	Neumann
Panel	Source/sink and doublet sheets	Dirichlet and/or Neumann

The LLT

One linear horseshoe vortex shedding it's strength incrementally in the wake



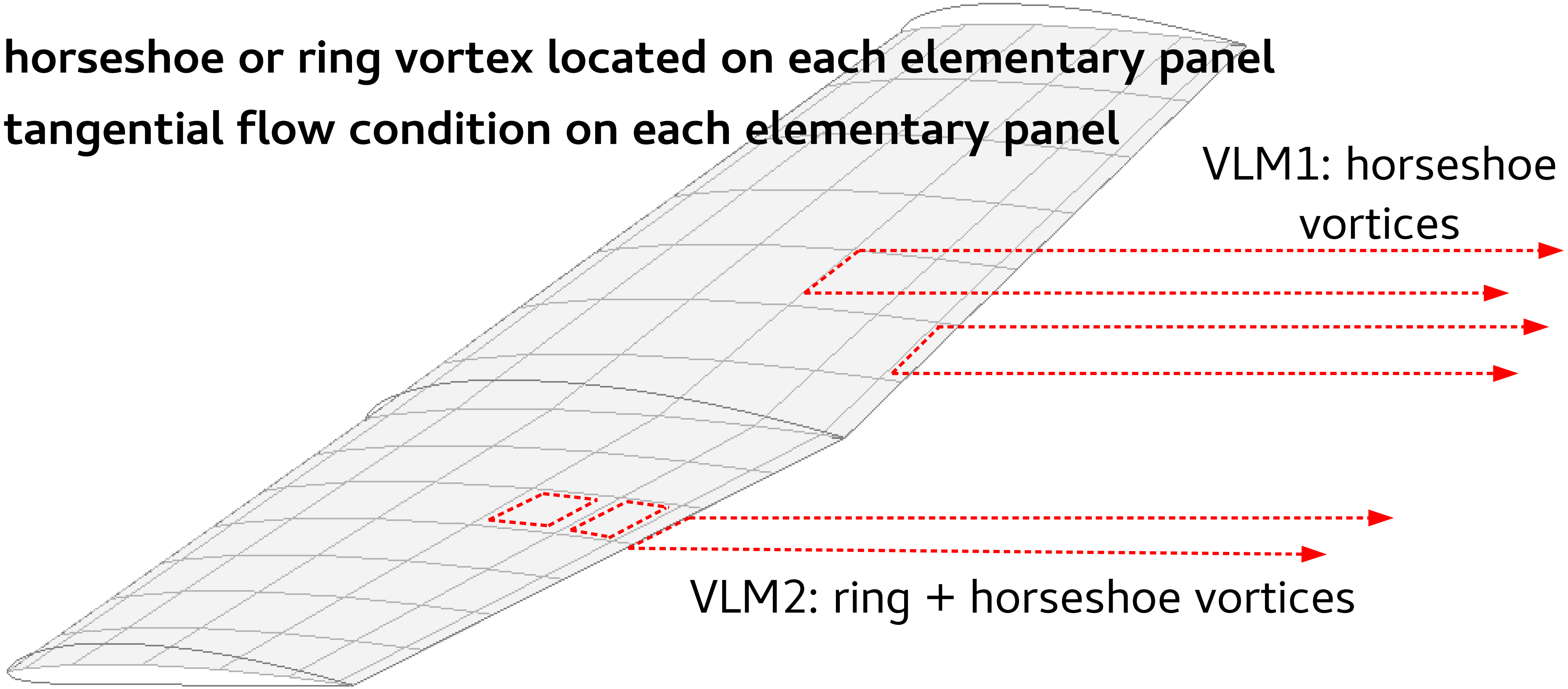
One linear horseshoe vortex → wing only, no dihedral, no sweep

xflr5 implements the non-linear LLT described in report NACA TN-1269

The VLM

One horseshoe or ring vortex located on each elementary panel

One tangential flow condition on each elementary panel



Highly versatile

Multiple lifting <u>thin</u> surfaces

The VLM

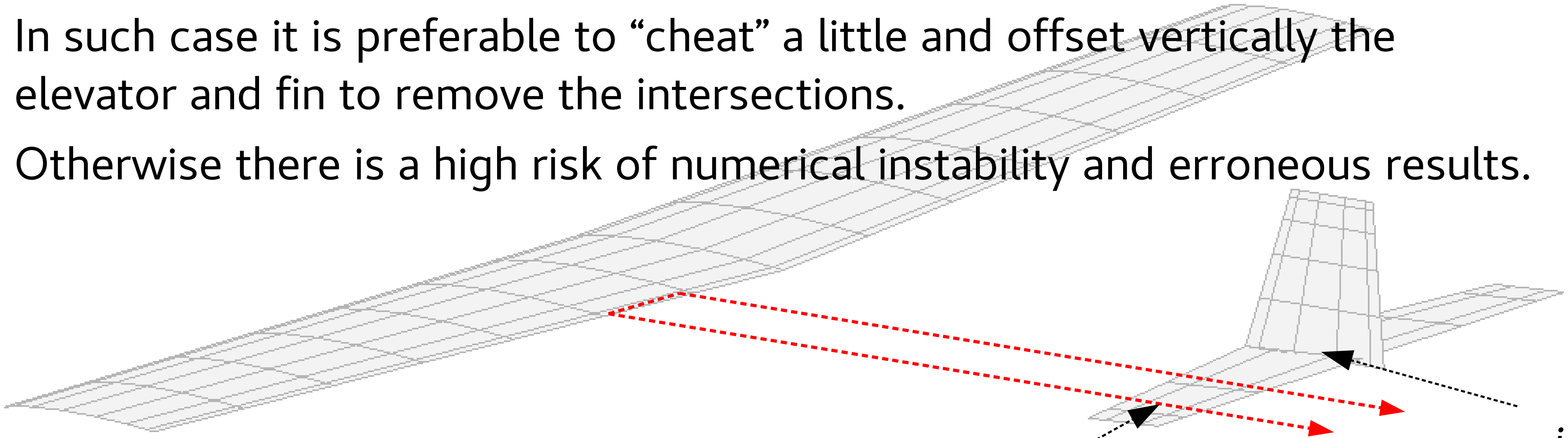
Main limitation of the VLM:

→ the wing's trailing vortices must not intersect the elevator and fin vortices

→ the wing panels should not intersect

In such case it is preferable to "cheat" a little and offset vertically the elevator and fin to remove the intersections.

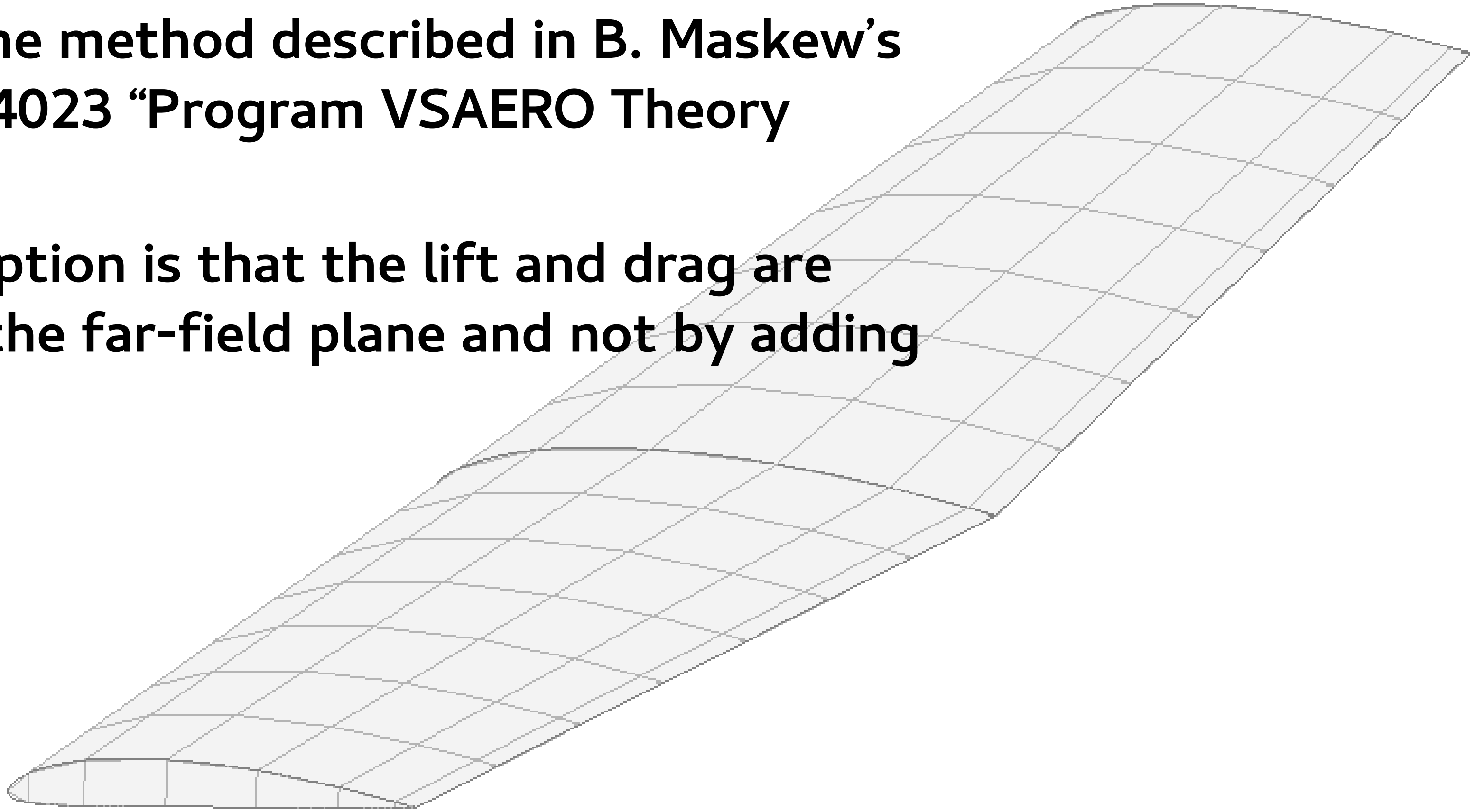
Otherwise there is a high risk of numerical instability and erroneous results.



The Panel method

Implements the method described in B. Maskew's report NASA 4023 "Program VSAERO Theory Document".

The only exception is that the lift and drag are calculated in the far-field plane and not by adding panel forces



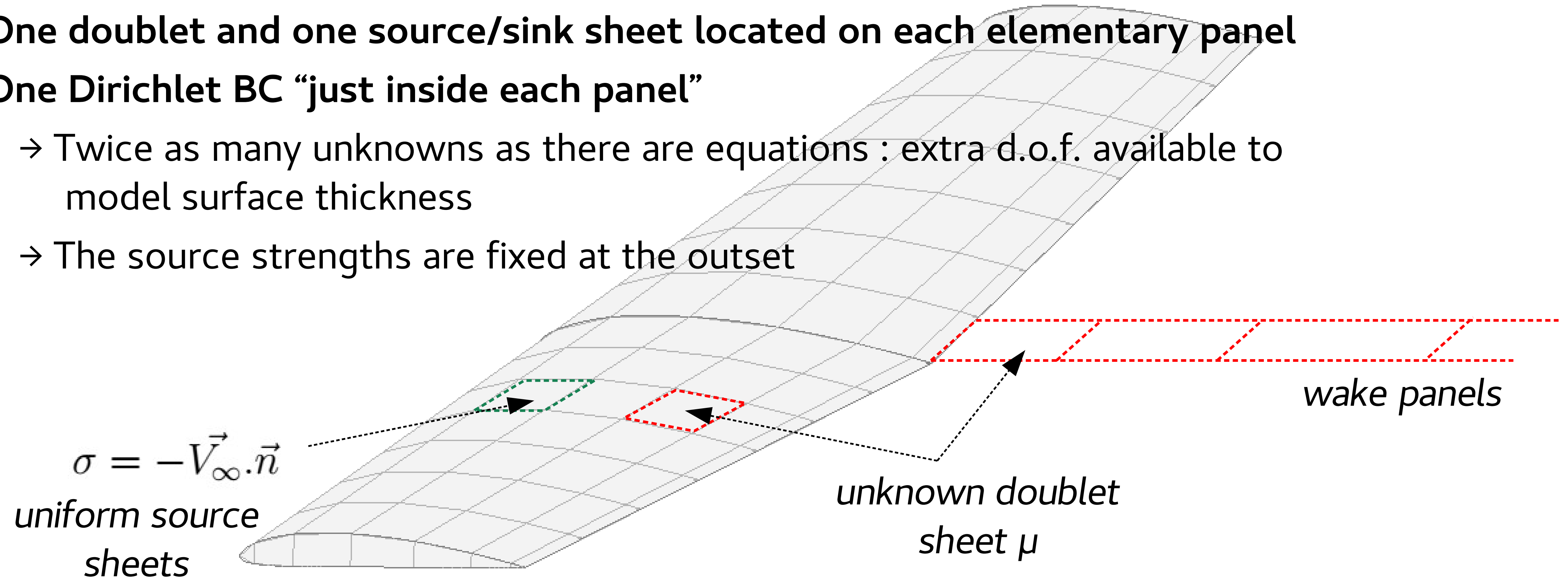
The Panel method

One doublet and one source/sink sheet located on each elementary panel

One Dirichlet BC "just inside each panel"

→ Twice as many unknowns as there are equations : extra d.o.f. available to model surface thickness

→ The source strengths are fixed at the outset



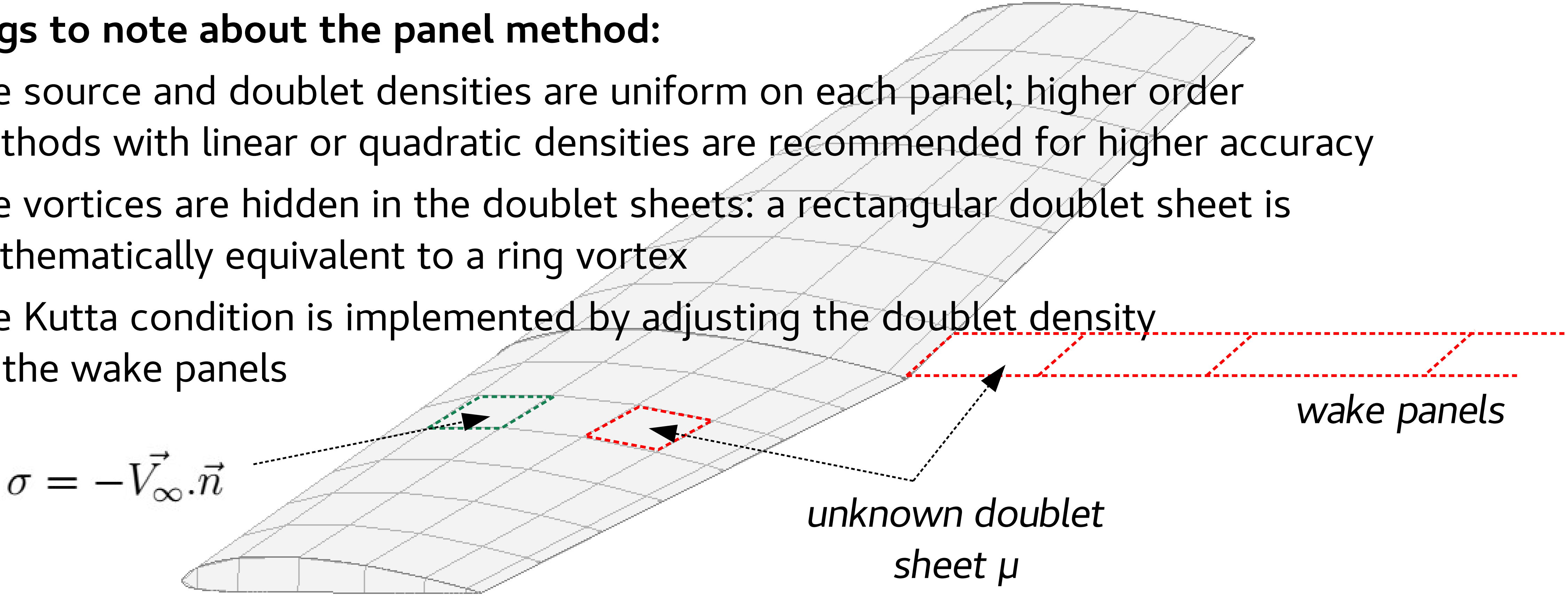
Highly versatile

Multiple lifting <u>thick</u> surfaces

The Panel method

Things to note about the panel method:

- The source and doublet densities are uniform on each panel; higher order methods with linear or quadratic densities are recommended for higher accuracy
- The vortices are hidden in the doublet sheets: a rectangular doublet sheet is mathematically equivalent to a ring vortex
- The Kutta condition is implemented by adjusting the doublet density on the wake panels



Highly versatile

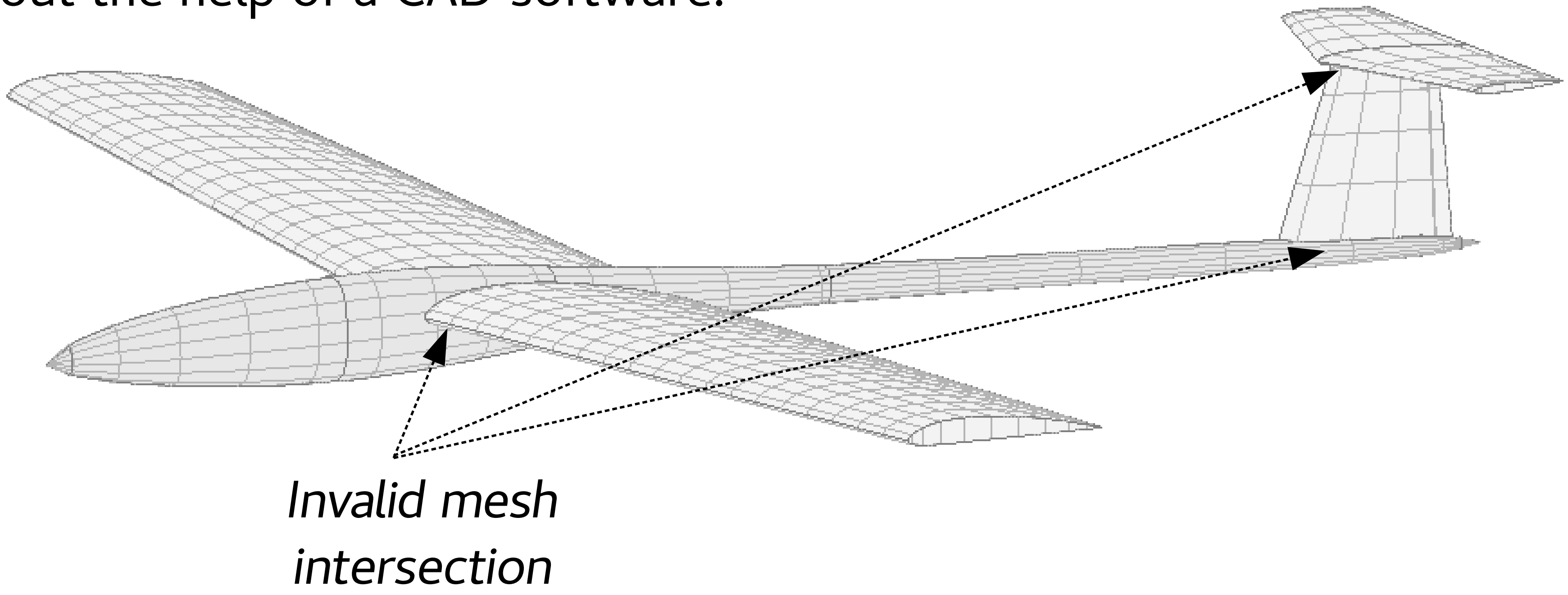
Multiple lifting <u>thick</u> surfaces

The Panel method

Limitation of the the panel method:

- same limitation as for the VLM
- + all volumes should be closed and non-intersecting

This second limitation is the reason why the thick wing option is disabled in xflr5 v6: wing, fuselage, elevator and fin cannot be merged or connected without the help of a CAD software.



Solving the problem

The linear system

Whatever the method, the problem is reduced to solving a linear system

$$\begin{bmatrix} & & \\ & a_{ij} & \\ & & \end{bmatrix} \begin{bmatrix} \mu_0 \\ \mu_1 \\ \cdots \\ \mu_{N-2} \\ \mu_{N-1} \end{bmatrix} = \begin{bmatrix} RHS_0 \\ RHS_1 \\ \cdots \\ RHS_{N-2} \\ RHS_{N-1} \end{bmatrix}$$

The a_{ij} coefficients are the potential influence of one panel on another.

In the case of the non-linear LLT, a viscous loop is added until convergence.

The last thing to do is to calculate the lift and drag from the source & doublet densities or vortex circulations.

Solving the problem

Induced lift and drag

The most straightforward method is to sum the forces acting on each panel. Over time however, numerical tests have shown that this method is imprecise.

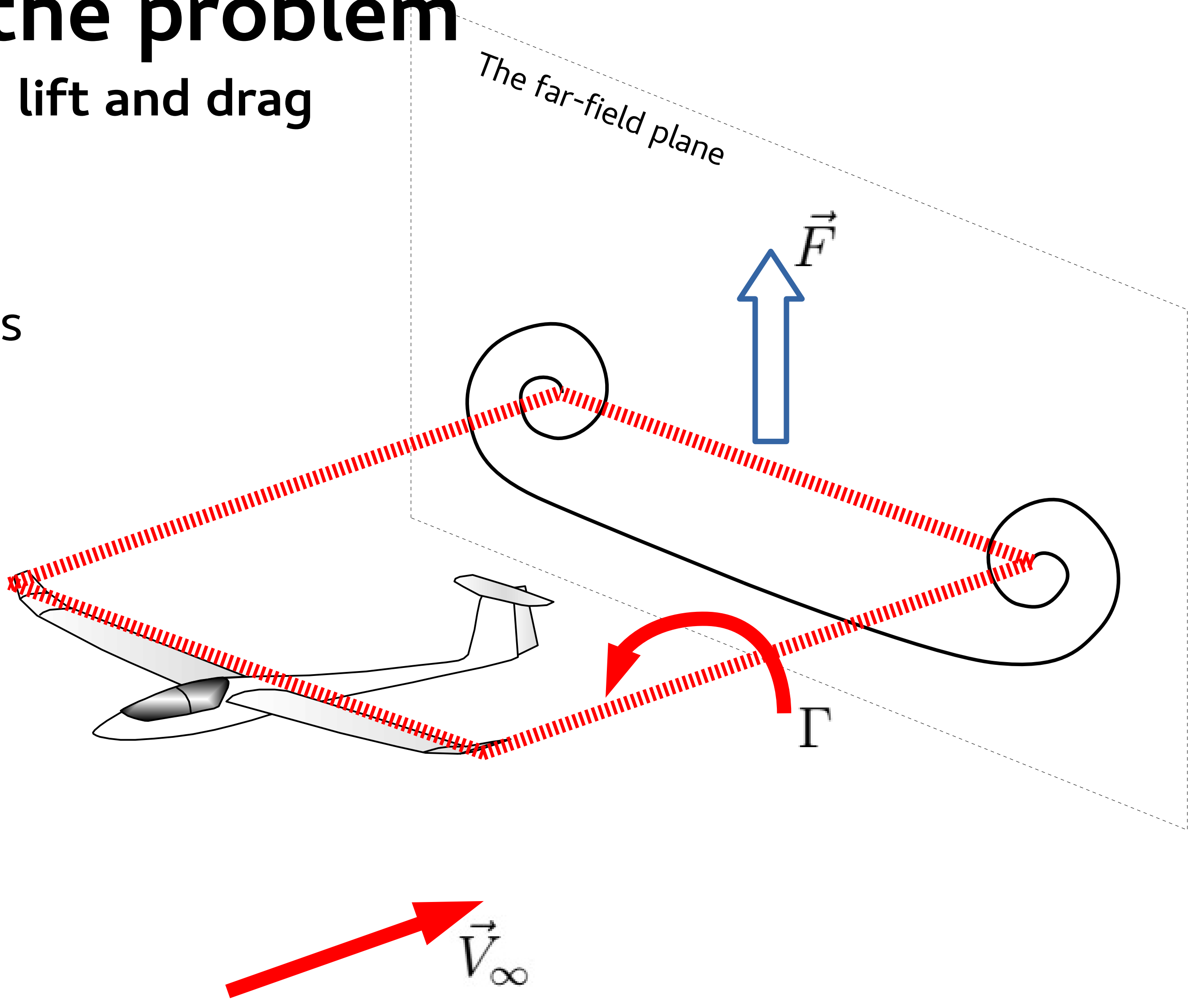
The preferred method is to determine lift and drag in the far-field Trefftz plane. Since

i. the vortex must form a closed filament

ii. the circulation is constant along the vortex

the lift on the wing is the lift in the FF plane calculated using Kutta-Joukowski's theorem.

This is recommended for panel methods in general and is implemented by default in xflr5



Solving the problem

Flow chart of a potential flow analysis

Discretize the the geometry

↓

Define the source strengths and the BC

↓

Write and solve the linear system, including the Kutta condition

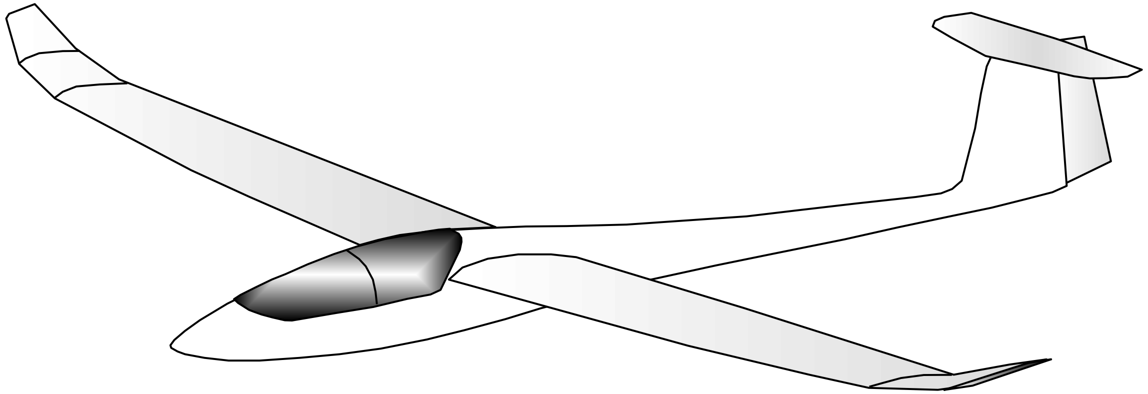
↓

Calculate lift, drag and other quantities of interest

↓

Proceed to the Boundary-Layer analysis

BL analysis is the object of the next presentation



- up next -

Why does an airfoil drag: the viscous problem